P8 – 0110 – 00039S
Wing Spar calculations

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1 SUMMARY

This report contains PIK28 wing spar calculation spreadsheet description.

Spreadsheet referred has ref points, in spreadsheet in red colour. They are pointers for the description here.
Cells that are intended for user input are yellow. Only they should be changed if you are just using it. Other cells contain formulas and should not be changed (unless you find error. If so, tell also to us).

2 BASIC DIMENSIONS

Sheet “Dim” contains basic wing geometry at three Y locations.

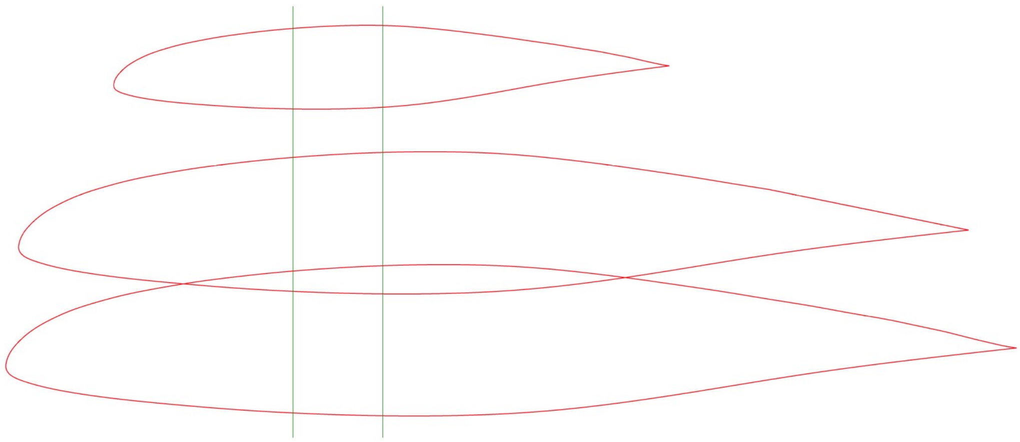
Note that root chord need not to be at aeroplane centerline. Values that are inside of root chord are extrapolated.

Tip Y location is used for wing span dimension. So it is important.

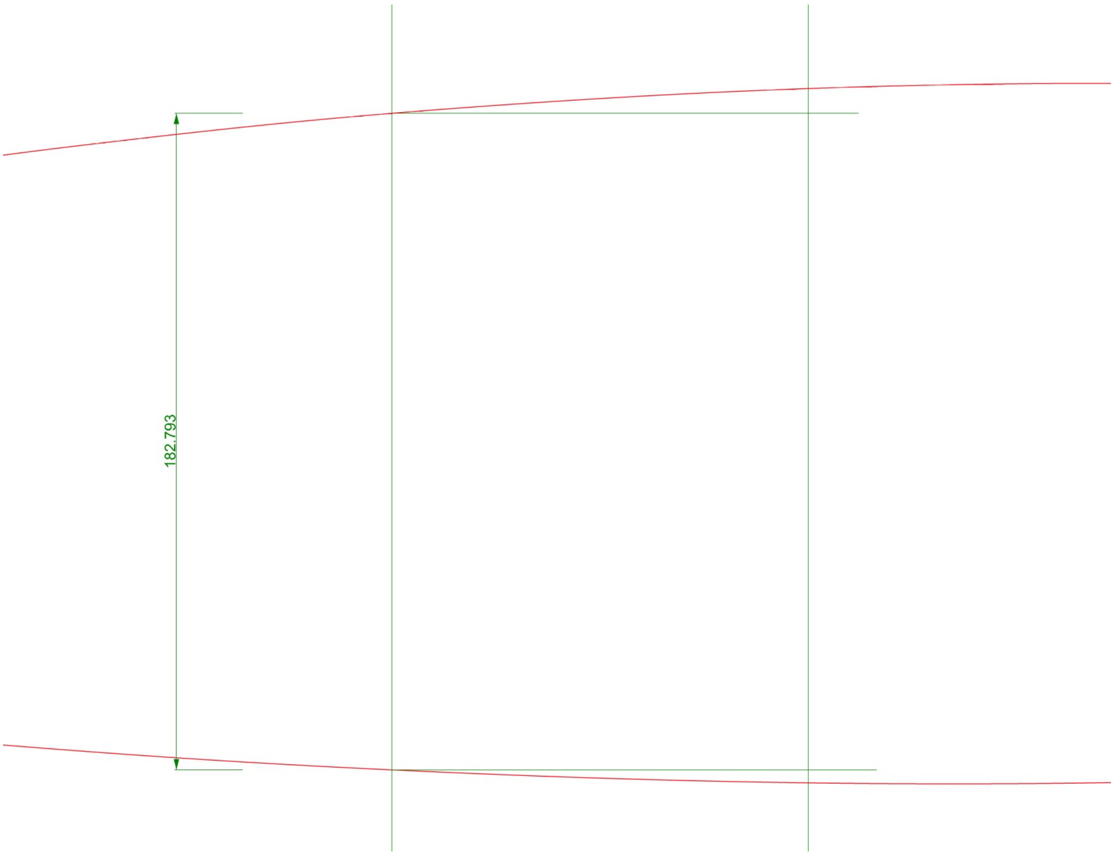
Max profile thickness is what is given in profile data as maximum thickness, this is used for cross-checking only. This might not be where spar box is planned, so this can not be used for spar box dimensions.

Spar box location is the value used extensively.

Picture shows profiles at three locations (root, mid, tip) and planned spar box location of Pik28, green lines showing front and rear edges.



Easily visible is that the thickest part of this 15% thick profile is aft of the spar box. This picture uses same width for spar box in every location. It can be variable, then intermediate locations are interpolated.
The meaning of thickness at spar location (ref #1) is illustrated below. You have to measure distances and convert them to % (of c) values, because calculations uses this. In this location chord length is 1305,8 mm.



First determine which corner is the slimmest. In picture above at top the front side of box is lower and also at bottom the front side is upper. It can be otherwise if profile is cambered. For calculation a square sparbox is used.

This dimension 182,79 mm is used to calculate spar box height, which in this case is 182,79 / 1305,8 * 100% = 13,998 %. This value is inserted into calculation sheet (ref #1).

If you use cover sheets that are used over sparbox, the thickness of this sheet (note: in millimeters!) is inserted right of the (ref #1) height value. In picture below the blue line. Same thickness is used for top and bottom. If thickness is varied, values are interpolated. So use carefully if final results are at edge.

Width of spar box can be variable. Width is interpolated between these three Y locations. In Pik28 spar has constant width, for simpler manufacturing. A width of 100 mm is because that is the common tape width for carbon and glass unidirectional reinforcement.

Give Y locations and corresponding width in [mm]. Ref #2. These can be different to wing geometry Y locations. Width is interpolated between these three locations.

From these values wing local chord values and spar box dimensions are calculated.

Calculated values are below row 24 of this sheet. They are referred in many other sheets.

3 DISTRIBUTED WEIGHT OF WING

Sheet "DisM". This sheet contains estimates of distributed mass of the wing. They are not calculated values, so you have to use other sources for suitable values. If in doubt, use underestimation for this.

Calculating locations in Y direction. Ref #3 is the number of divisions in Y direction. Do not change this (46)! As it does not change the number of rows used. If you change, be prepared to change number of rows in every sheet!

Ref #4 has distributed mass. First Structure mass (columns C-D)
Upper row has Y location (root) and linear mass at root kg/m.
Lower row has tip Y location and the linear mass value in kg/m. You have to guess these estimates. Not calculated (yet).
See that graph below how inserted values affect wing distributed weights.

Next to right is fuel location and volume in liters. It uses density of 0,75 kg/liter (car fuel). Change density to 0,72 (in the formula) for Avgas, or whatever you like.
There is two other weight locations. If you want point loads, you have to cheat a bit. Insert total point mass on upper yellow cell. Then in cells (from to) location so that in cells below only two row has values, it will be the linear mass calculated from inserted values. Check from total wing weight (ref #5) that the sum is correct.

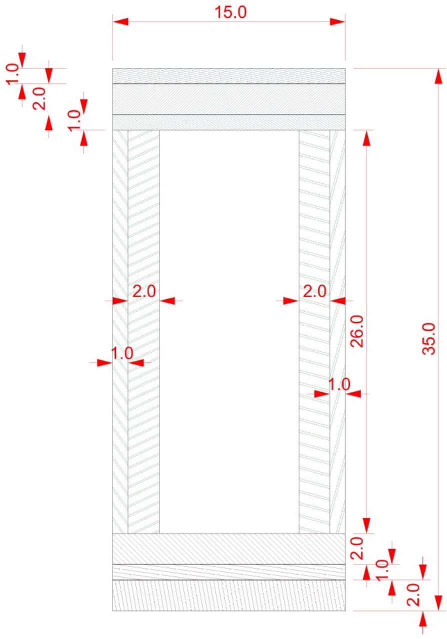
Location of stay. Removed from this calculation spreadsheet. This calculates only cantilever wing.

4 SPAR

Sheet "Spar".
Matrix for materials used is in area ref#7. For each line first column is index number. Only values 1 to 10 are used.
Material values are in pascals (Pa), so that no multiplier is needed in any formulaes.

Area ref#8 is the design area. Top (and bottom) spar caps can be made of 11 layers. Web can be made of four layers.
For each layer for no material, safest is give reasonable start and end Y coordinates, and zero for thickness.
For partxx, give where that material is starting and ending. Also given are thickness at root and tip, intermediate locations are interpolated.

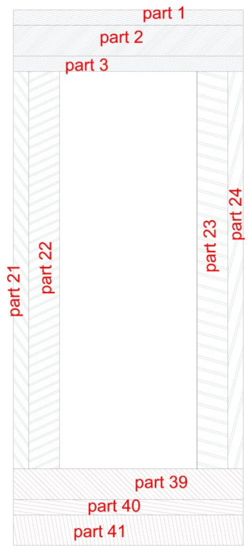
Material index is given in column G.
For showing how calculation proceeds, following layout is used:



Here only three layers on bottom and on top are used. For web symmetrical 2+ 2 is used. Main dimensions are shown in picture.
For this sample, E values of:
for the 1mm thick layers; E = 1,18E+10 (i.e, plywood 0/90)
for the 2 mm thick layers; E = 3,40E+10 (i.e, UD glass tape)
web layers E = 2,70E+09 (i.e, plywood 45/45)

Starting from (#ref9), each part is calculated in one block surrounded by thick borderline.

These are the numbers of parts in the sample:



First part (ref#9) is the bottom spar first layer (part41).
At top of this border is “Modulus of elasticity” in tension for this material. Calculation is for positive g load, so this bottom spar is in tension.
First column (logical result) check if at this Y location material layer exists (if you enter sample case there is TRUE). If test looks at the end of this step, so if material (#ref8) ends in this step, then it is not included.
Next (column to right) is thickness (Z coord) for this layer. Next is Area*Modulus.
In the example:

$$0{,}015 \times 0{,}002 \times 3{,}40E{+}10 = 1{,}020E{+}6$$

This is done to normalize different materials with. In hand calculation usually ratio $E_{local}/E_{referense}$ is used. As reference here a value of 1 is used.
Next is local section Area moment of inertia (I loc = $bh^3/12$).

$$\frac{0{,}015 \times 0{,}002^3}{12} = 1{,}0E{-}11$$

This is I for this layers abound it’s own neutral axis. Later when total box neutral axis is calculated, the next two columns can be calculated.
Where Steiner term is distance from neutral axis to power of 2 times local area (times E). This is to calculate moment of inertia with respect to arbitary axis.
From later calculations, neutral axis is calculated to be 38,5 mm from bottom surface. So distance from neutral axis to center of this layer is

$$0{,}0385-\frac{0{,}002}{2}=37{,}5$$

And Steiner term times modulus term for this layer is;

$$0{,}0375^2\times1{,}020\text{e+}6=1{,}434\text{e+}03$$

Note: Spreadsheet gives for this as result = 1,44E+03. This is due for higher count of decimals used in its calculations. Like the neutral axis distance is 3,8539e-2 m.

Last column is local area moment of inertia times modulus.

$$1{,}0\text{E-}11\times3{,}4\text{E+}10=3{,}4\text{E-}1$$

In following boxes (part40, part39, etc) there is in first column the distance from spar box bottom to the bottom of that layer. This is not present in first box as it is the first layer.

Starting from (ref #10) are similar calculus for vertical members (shear web). Parts 21 … 24.
Next (ref #11) is same calculus as for lower side spar caps but for upper side spar caps. This means that modulus in compression is used. And distance from top surface is calculated.

Similar to (ref#7), sum of EA and EA*dist are calculated (ref#8).

item	Modulus E	Area A [m²]	Area moment of inertia I_{loc} [m⁴]	A * E	Center dist from bottom level t [m]	A *E * t	
Part 41	3,4e+10	3e-5	10e-12	1,02e6	0,001	1,02e3	
Part 40	1,18e+10	1,5e-5	83,33e-15	177e3	0,0025	443	
Part 39	3,4e+10	3e-5	10e-12	1,02e6	0,004	4,08e3	
Part 1	1,18e+10	1,5e-5	83,33e-15	177e3	0,0345	6,11e3	
Part 2	3,4e+10	3e-5	10e-12	1,02e6	0,033	33,7e3	
Part 3	1,18e+10	1,5e-5	83,33e-15	177e3	0,0315	5,58e3	
Part 21	2,7e+9	2,6e-5	1,465e-9	70,2e3	0,018	1,26e3	
Part 22	2,7e+9	5,2e-5	2,929e-9	140,4e3	0,018	2,53e3	
Part 23	2,7e+9	5,2e-5	2,929e-9	140,4e3	0,018	2,53e3	
Part 24	2,7e+9	2,6e-5	1,465e-9	70,2e3	0,018	1,26e3	
		Sum of		4,01e6	Sum of	58,5e3	

Neutral axis position (from bottom) is
sum of EA*dist / sum of EA

58,5e3/4,01e6 = 1,46e-2 = 14,6 mm

spreadsheet gives same result.

All three components (bottom spar laminate), (top spar laminate) and (webs) are summed and (ref#10) neutral axis is calculated.

This neutral line location is used in each part to calculate Steiner term value.

item	A * E	Center dist from bottom level t [m]	Dist from neutral axis t_{na}	A*E * t_{na}^2
Part 41	1,02e6	0,001	0,0136	188,66
Part 40	177e3	0,0025	0,0121	25,91
Part 39	1,02e6	0,004	0,0106	114,61
Part 1	177e3	0,0345	0,0199	70,09
Part 2	1,02e6	0,033	0,0184	345,33
Part 3	177e3	0,0315	0,0169	50,55
Part 21	70,2e3	0,018	0,0034	0,81
Part 22	140,4e3	0,018	0,0034	1,62
Part 23	140,4e3	0,018	0,0034	1,62
Part 24	70,2e3	0,018	0,0034	0,81
			Sum of	830,01

local area moment * E are summed

item	Modulus E	Area moment of inertia I_{loc} [m^4]	E * I_{loc}
Part 41	3,4e+10	10e-12	0,34
Part 40	1,18e+10	83,33e-15	0,001
Part 39	3,4e+10	10e-12	0,34
Part 1	1,18e+10	83,33e-15	0,001
Part 2	3,4e+10	10e-12	0,34
Part 3	1,18e+10	83,33e-15	0,001
Part 21	2,7e+9	1,465e-9	3,96
Part 22	2,7e+9	2,929e-9	7,91
Part 23	2,7e+9	2,929e-9	7,91
Part 24	2,7e+9	1,465e-9	3,96
		Sum of	24,76

Knowing neutral line, finally total area moment of inertia around this neutral line can be calculated. (ref#11). Which is sum of each parts Steiner term and local moments of inertia.
830,01 + 24,76 = 854,77

spreadsheet gives 8,25e+2, and looking from intermediate results, the difference is due to rounding errors in this hand calculations.

The same calculus is repeated for each location on next rows.

Results are shown in two graphs;
First showing spar box total geometric height and neutral axis height from bottom.
A location that is near mid height is reasonable design.
The other graph shows the total EI value at different locations of the wing.

5 STRAIN CALCULUS

Having EI we can calculate strain when load are known.
Load definition starts with aircraft V-n diagram. (sheet “V-n diagram”) where aircraft flight envelope is defined.
Wing mass distribution is given in sheet “MassDist”. Note that this is not using mass calculations from sheet “Mass”, which calculates mass of defined structure. So you have to move calculated mass to this estimate couple of times. Making it automatic creates easily a divergent solution. This method (although more manual work) is stable.
You are advised to start with lower than actual mass, which is conservative.

6 SHEAR CALCULUS

Shear stresses vary in a given crosssection.
Basically shear stress is:

$$\tau = \frac{V \cdot Q}{I \cdot t}$$

where:
V = shear force
Q = first moment of area (above/below location)
I = moment of inertia (to neutral axis)
t = width at the location where shear is calculated

As we have different materials, we calculate shear strain, not shear stress. For relation between shear stain and shear stress, we need G (glide modulus) of the material:

$$\tau = \gamma \cdot G$$

where:
τ = shear stress
γ = shear strain
G = glide modulus

By dividing the shear stress formula with G we get:

$$\frac{\tau}{G} = \frac{V \cdot Q}{I \cdot t \cdot G}$$

$$\gamma = \frac{V \cdot Q}{I \cdot t \cdot G}$$

For a given (distance from neutral axis) first moment of area needs:
A= is area above the point of interest.
B= A x distance from center of area to neutral axis.

$$Q = A \times B$$

When calculating shear stress in given section, like between parts 1 and 2 of our example, we need:
Q = for part 1 / G

$$Q_1 = A \times B = (15\ mm \times 1\ mm) \times (\text{height} - \text{half of part 1 thickness} - \text{neutral axis from bottom})$$
$$= (15\ mm^2) \times (35\ mm - 1\ mm/2 - 14{,}6\ mm)$$
$$= 15 \times 19{,}9\ mm^3 = 298{,}5\ mm^3$$

From material data we get that for part 1 (plywood 0/90) G is 1E+9 Pa. So the
$Q/G = Q_1/1E+9 = 2{,}985E-16$.

T = width = 15 mm
I = total moment of inertia

Total moment of inertia need to be calculated with multiplying of the material G modulus.

https://www.youtube.com/watch?v=Ibs30_hnSpA

---- END ----